2: Qubits

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We represent polarization states with vectors

orthonormal basis { ↕ ~ $\begin{pmatrix}1\\0\end{pmatrix}$ = $|0\rangle$ "zero state" "ket zero" } "standard basis states"
↔ ~ $\begin{pmatrix}0\\1\end{pmatrix}$ = $|1\rangle$ "one state" " one "

(Ket notation)

o.n. basis { ⤢ ~ $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ = $|+\rangle$ "plus state" ket plus
⤡ ~ $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ = $|-\rangle$ "minus state" ket minus

o.n. basis { ~ $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ clockwise
~ $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ counter-clockwise

Note $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$

How many qubit states are there?

A) 2. B) 6 C) Countably infinite D) Uncountably infinite ↑

θ ~ $\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$

Qubit State

$\begin{pmatrix}a_0\\a_1\end{pmatrix} \in \mathbb{C}^2$ $a_0, a_1 \in \mathbb{C}$, $|a_0|^2 + |a_1|^2 = 1$

↑ "amplitudes" ↑ "normalization condition"

$aa^* = |a|^2$

	Linear algebra	Quantum state
Arbitrary Vector:	$\vec{v}$	$\lvert\psi\rangle$ ψ = psi = "sigh" "ket/state psi"

$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$ means $|\psi\rangle = \begin{pmatrix}a_0\\a_1\end{pmatrix}$

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

	L. A.	Q. M
Conj transpose	$\vec{v}^\dagger$	$\langle\psi\rvert$ ↑ bra psi

ex: $\langle 0| = (1 \quad 0)$

$\langle \circlearrowright | = \frac{1}{\sqrt{2}}(\langle 0| - i\langle 1|)$

	L. A.	Q. M
Inner Product	$\vec{v}^\dagger\vec{u}$ matrix multiplication	$\langle\psi_0\lvert\psi_1\rangle$ abbreviation of $\langle\psi_0\rvert\lvert\psi_1\rangle$

Q: If $|\psi\rangle$ is a qubit state, what is $\langle\psi|\psi\rangle$?

A) 0 B) $\frac{1}{2}$ C) 1 ↑ D) Depends on state

$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = a_0\begin{pmatrix}1\\0\end{pmatrix} + a_1\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a_0\\a_1\end{pmatrix}$

$\langle\psi| = a_0^*\langle 0| + a_1^*\langle 1|$

(← implied matrix multiplication)

$\langle\psi|\psi\rangle = (a_0^*\langle 0| + a_1^*\langle 1|)(a_0|0\rangle + a_1|1\rangle) =$
$= a_0^* a_0\langle 0|0\rangle + a_1^* a_0\langle 1|0\rangle + a_0^* a_1\langle 0|1\rangle + a_1^* a_1\langle 1|1\rangle$

$|0\rangle, |1\rangle$ are orthonormal basis!

$\langle 0|0\rangle = 1$ $\langle 1|1\rangle = 1$
$\langle 0|1\rangle = 0$ $\langle 1|0\rangle = 0$

$= a_0^* a_0 + a_1^* a_1 = |a_0|^2 + |a_1|^2 = 1$

Remember: $|a_0| = \sqrt{a_0^* a_0}$ so $|a_0|^2 = a_0^* a_0$

Same meaning of normalized as usual!

→ "$|+\rangle$ is a superposition of $|0\rangle$ and $|1\rangle$"

"Superposition" = linear combination (usually of standard basis states)

Measurement

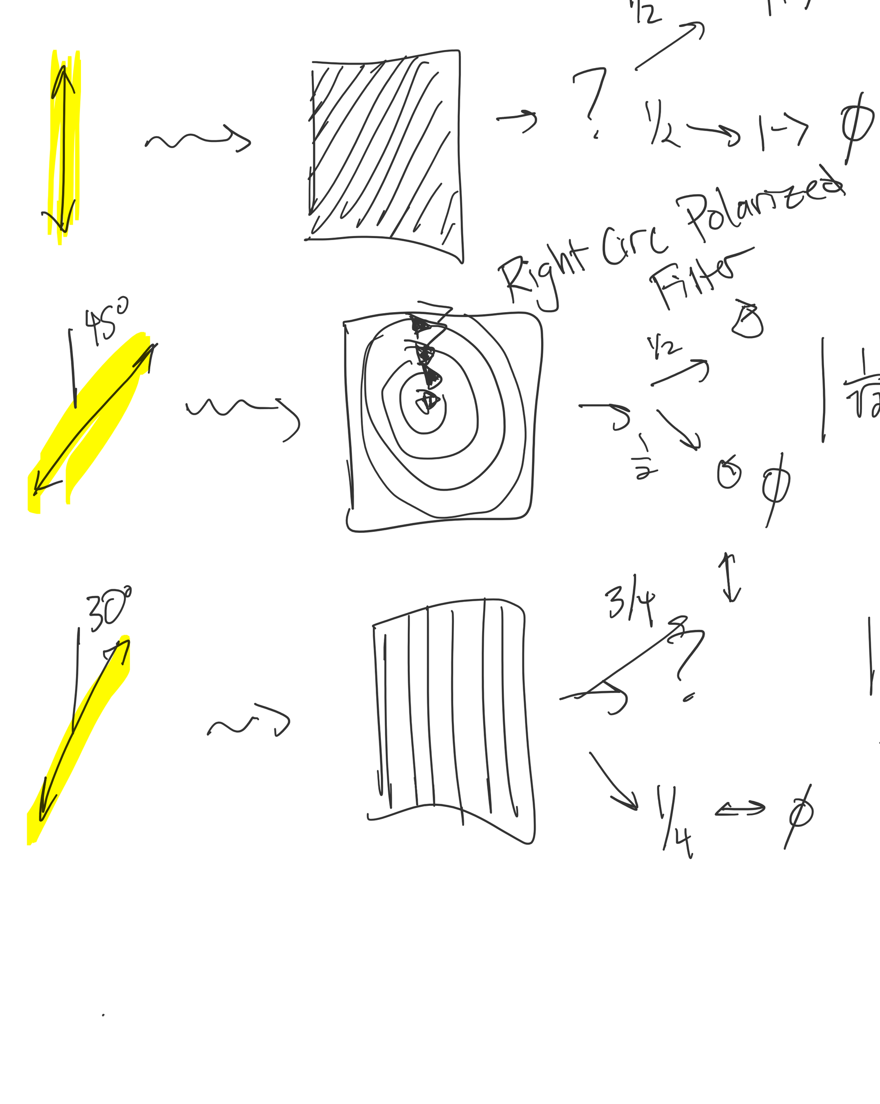
Represented by an orthonormal basis: $M = \{|\phi_1\rangle, |\phi_2\rangle\}$ "phi"

If measure state $|\psi\rangle$ with measurement M
- with probability $|\langle\phi_0|\psi\rangle|^2$, get outcome $|\phi_0\rangle$, state becomes $|\phi_0\rangle$
- with probability $|\langle\phi_1|\psi\rangle|^2$, get outcome $|\phi_1\rangle$, state becomes $|\phi_1\rangle$

↑ absolute value squared of inner product — measurement collapse

$|\langle\psi|\phi\rangle| = |\langle\phi|\psi\rangle|$

Describe/Analyze Using kets/bras + linear algebra



$|0\rangle$ measured with $\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$

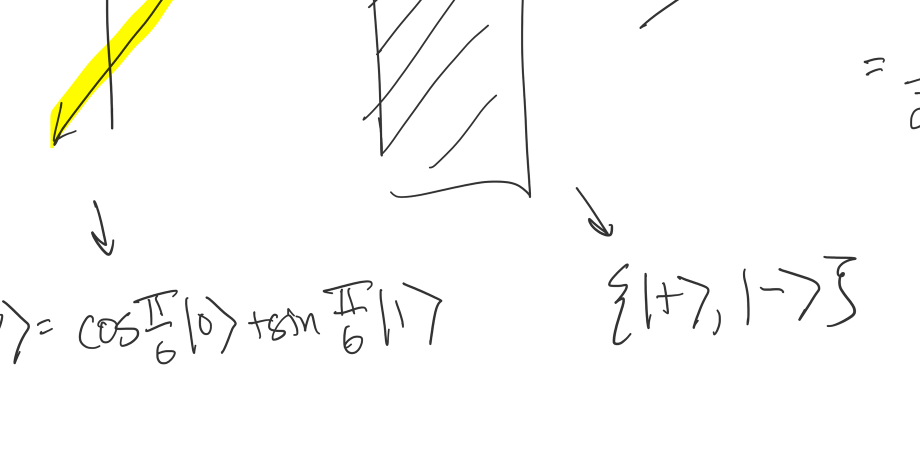
$|\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)|0\rangle|^2 = |\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$

$|\frac{1}{\sqrt{2}}(\langle 0| - i\langle 1|)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|^2 =$
$|\frac{1}{2}(1-i)|^2 = \frac{1}{4}(1-i)(1+i) = \frac{1}{4}(1 - i + i + 1) = \frac{1}{2}$

$|\langle 0|(\cos 30°|0\rangle + \sin 30°|1\rangle)|^2$
$= |\cos 30°|^2$
$= |\sqrt{\tfrac{3}{4}}|^2 = \frac{3}{4}$

- Measurement is rule we've observed

Ex:



$|\psi\rangle = \cos\frac{\pi}{6}|0\rangle + \sin\frac{\pi}{6}|1\rangle$ $\{|+\rangle, |-\rangle\}$

$|\langle +|\psi\rangle|^2 = \left|\left(\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)\right)\left(\cos\frac{\pi}{6}|0\rangle + \sin\frac{\pi}{6}|1\rangle\right)\right|^2$

$= \frac{1}{2}\left|\cos\frac{\pi}{6}\langle 0|0\rangle + \cos\frac{\pi}{6}\langle 1|0\rangle + \sin\frac{\pi}{6}\langle 0|1\rangle + \sin\frac{\pi}{6}\langle 0\,1\rangle\right|^2$

$= \frac{1}{2}\left|\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\right|^2$

$= \frac{1}{2}\left|\frac{\sqrt{3}}{2} + \frac{1}{2}\right|^2 = \frac{1}{8}(\sqrt{3}+1)^2 =$

$\sim .93$